On Measuring Overall Well-Being from distributions of Capabilities*

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1 de julio de 2004

Abstract

In the debate to arrive at a measure of well-being, Amartya Sen put forward the space of capabilities as the setting of discussion. It is in this space that informational bases are capabilities; that is, substantive freedoms of individuals to promote values and to live the kind of life dictated by those values. For the implementation of well-being measures, it is generally required to restrict the domain of all possible dimensions of well-being to a finite number of them. In the space of capabilities, more than one capability are generally proposed to approach the relevant dimensions of well-being. In order to arrive at a measure of well-being, aggregation rules over multidimensional spaces generally require that principle of independence among dimensions. Since we accept that there is relevant complementariness and interconnections among capabilities, there is no chance to accept this independence. The common solution to dependence consists on establishing a relationship from capabilities to a neighbour space (commodities, utilities). We define this relationship to space of probabilities. From a combination of a dominance binary rule and the lexicographic

*I am grateful to Jorge Alcalde-Unzu, Miguel A. Ballester, Izete Bagolin, Flavio Comim, Monimoya Sengupta for reading an earlier draft of this paper. I also wish to thank to the research group SEED background of the earlier endeavours. The project is supported by Spanish Government (CICYT: SEC [2003-08105]), by Universidad Pública de Navarra and was developed during my stay at Capability and Sustainability Centre, St. Edmund's College, University of Cambridge.

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order defined with information from both, space of capabilities and space of probabilities, we define an order of profiles of capabilities.

Keywords: Capabilities Approach, Functioning, Well-Being, Aggregation rules.

JEL classification:

1. Introduction

In the debate to arrive at a measure of well-being, Amartya Sen(1985) put forward the space of capabilities as the setting of discussion. It is in this space that informational bases are 'capabilities' (substantive freedoms) of individuals to promote values and to live the kind of life dictated by those values. Hence, as far as well-being is measured and evaluated in terms of capabilities, well-being is directly related to the ability of individuals to achieve valuable features of life. The features of life are named 'functionings' in Sen's terminology. Indeed, neither commodities, nor opportunities provide well-being by themselves. Those instruments provide well-being to individuals as far as they enable people to achieve functionings.

From the analysis of well-being in spaces such as the space of commodities, it can be assumed that person 'A' enjoys more well-being than person 'B', if A has the right to use a bicycle, and 'B' does not. There is no chance however to support this assumption in the space of capabilities. The possession of the bike increases well-being as far as the bike enables people to achieve functionings. In the case in which person A can not use the bike (given a physical disability, for example), and neither can she exchange it to achieve any functioning, the possession of bike does not increase her well-being. Well-being will be increased only as far as she has the chance to use it and therefore, to achieve the ability of movement (or to exchange it obtaining resources that provide her some functionings).

For the implementation of well-being measures, it is generally required to restrict the domain of all possible dimensions of well-being to a finite number of them. To approach many of the relevant dimensions, authors such as

Nussbaum (2000) list a finite number of capabilities. Alkire (2002) summarises lists like this, which are often defined in the space of needs. Instead of an explicit list of capabilities, Sen supports and proposes the definition of categories of capabilities. In particular, he defines five categories of capabilities in order to capture the most relevant types of instrumental freedoms which enable individuals to live well. The five categories are: 'political freedoms', 'economic facilities', 'social opportunities', 'transparency guarantees' and 'protective security'.

If an explicit list of capabilities is required or a list of categories of them, a measure of well-being needs agree those relevant points to have well-being measured. In the space of capabilities, more than one capability are generally proposed to approach the relevant dimensions of well-being. We name *sub*-capabilities to each one of these capabilities. We consider that these *sub*-capabilities approach the general capability of individuals to achieve well-being.

Capability of a person and her *sub*-capabilities can be identified, but (because they are freedoms) it is not very easy to have them measured. To facilitate the measure of capabilities, the expansion of someone's capability is related to her ability to chose among combinations of functionings, Sen(1985). Combinations of functionings are observable features of human lives. While large discussion can arise before arriving at a measure of the freedom of movement someone has. The measure seems to come more straight when the objective things that she can really do are measured (for example, the distance and frequency with which she can move).

In order to have capabilities (and *sub*-capabilities) gradually defined, some authors such as Chiappero-Martinetti(1996) proposes a dichotomic measure. Using the real numbers $\{0, 1\}$, she identifies with number 1 those functionings any of which an individual i may realize, and with number 0 those functionings that person i can not realize. Our proposal defines gradual *sub*-capabilities using the numbers in the real line; to simplify, we develop the exercise in the interval [0,1] in the real line.

Beyond having *sub*-capabilities gradually defined, and in order to measure their expansion, we compile for each *sub*-capability a set of functionings related to it. We assume that those functionings can be ordered on the basis of the ability to overlap the *sub*-capability. We also assume that there is a strictly increasing function per *sub*-capability defined to the interval [0,1] from its related order of functionings.

Apart from this, the approach of well-being can be carried out either: through the examination of information released in the realized combination of functionings or through the examination of that set of available combinations of functionings. On the one hand, well-being can be related to the combination of functionings that individuals realize. In this procedure, freedom is valued as an instrument which enable individuals to choose their best option, but it is not assumed to be a source of well-being. Hence, this procedure loss information about individuals' real freedom to live well. Indeed, it is not considered the well being generated from the intrinsic value of freedom. A familiar example illustrates this loss of information. Going on a fast (given certain moral reasons) and starving (given no alternative situation) may entail living a similar kind of life. That is, we can consider that they enjoy the same level of well-being. If intrinsic value of freedom is taken into account, a person with more freedom of choice should indeed be considered as enjoying more well-being.

For that case in which intrinsic value of freedom is supposed to provide well-being, well-being is related to the set of available combinations of functionings any of which can be chosen. This set is named 'capability set' in Sen's terminology. Certainly, our proposal to measure well-being is focused in the capability set of individuals, instead of the combination of functionings which they realize. Along the paper, we want to explore measures of the overall well-being in a society, in the space of capabilities. The aggregation of individuals' well-being to arrive at an overall measure has been largely discussed in alternative spaces such as that of utilities (Arrow (1951)), opportunities (Roemer (1996)). This is still an open debate, and it is especially in the space of capabilities.

Since we accept that there are relevant complementariness and interconnections among *sub*-capabilities, there is no chance to accept this independence. Certainly, dependence among capabilities is especially restrictive when arriving at a measure of well-being. Independence among dimensions is needed to apply common aggregation rules, such as the aggregation by weighting. Weighting entails the assumption that each *sub*-capabilities enhance human life, independent of each other. That is, weighting entails that human life is enhanced with a further guarantee of one of the capabilities, when there is a contraction in the guarantee of another capability.

Some solutions to weight are proposed in the space of capabilities. The common solution consists on establishing a relationship from capabilities to a neighbour space. For example, Desai (1990) uses the space of income and

Chakraborti (1996) the space of utilities. Their proposals are discussed in the following section. In section three, we introduce our proposal.

Finally, the paper is distributed as follows. In the next section, we propose a definition of gradual capability set and the definition of a dominance rule to rank capabilities sets. Along section three, we discuss the overall well-being measures proposed in the literature in the space of capabilities. In it, we propose our refinement of the dominance rule in order to get an order of profiles of capabilities. And, finally, we conclude in section four.

2. From Individual Well-Being to Overall Well-Being.

In order to have a measure of the overall well being provided by public policies, information is often aggregated through two procedures. Each one generally involves two steps. The first one consists of aggregating data to have an overall individual's well-being measure, and then, aggregating across individuals to obtain an overall well-being. The second is based in the opposite route: first, aggregating data over individuals to have an overall measure in guaranteeing each *sub*-capability and then, aggregating across *sub*-capabilities to have the overall well-being.

Dutta, Pattanaik and Xu (2003) discuss whether or not any aggregation rule is free of the path dependence problem. That is, whether, or not, any aggregation rule obtain the same measure of the overall social deprivation in case that this rule aggregates the overall individual deprivation first, and in case that this rule aggregates the overall deprivation for guaranteeing each capability first. They notice that whichever rule is not actually free of this problem of path dependence. They release a version of the following example. In their version, they refer to achievement of attributes, where we refer to the level in which capabilities are guaranteed.

Data for three individuals and for the level in which three capabilities are guaranteed are those, such that:

	Capabilities (Attributes)		
Individual A	0.33	0.33	0.00
Individual B	0.33	0.33	0.33
Individual C	0.00	0.00	0.33

It is path dependent a measure of the overall well-being based on the following two rules. The first one is the arithmetic mean. This is used to aggregate the overall deprivation in each attribute (vertically in our matrix). The second one is that rule proposed by UN Development Program's (1997). This rule consist on calculating the mean of the cube of the focal variables and then calculating the cube root of this result. This rule is used to aggregate the overall deprivation for each individual (horizontally in our matrix).

The overall deprivation of this society is 0.28 following the first procedure (first horizontally and after vertically), while this is 0.22 following the second one (first vertically and after horizontally).

Dutta, Pattanaik and Xu demonstrate that the simple aggregation rule based on *weighting* data is, in fact, path independent. As formerly we introduce, since we accept that there are relevant complementariness and interconnections among *sub*-capabilities, there is no chance to accept this independence. A solution consists of weighting in a neighbour space. Desai (1990) and Chakraborti (1996) propose some solutions in this line.

Desai's solution consists of establishing a correspondence between the space of capabilities and the space of income. In an attempt to compare well-being among individuals (to identify whom of them are poor), he makes use of that idea of 'poverty line'. An amount of money is defined to overlap capabilities. While a fixed amount of income defines the line to overpass in the general literature related to poverty line. This amount of income is personal in Desai's proposal. He introduces the fact that the amount of income needed to avoid deprivation depends (among other features) on her 'personal characteristics'. Somebody who suffers from a stomach disease probably needs more income to overlap the capability of living a healthy life than others who do not. This amount of income also depends on the 'others' living levels'. So, he includes the fact that the income that somebody need to overlap these capabilities is closely related to her neighbours' achievements.

Adam Smith (1776:470) exemplifies that case in which sources needed to guarantee the capability to appear in public without being ashamed depends on other's achievements. As he illustrates, while Greeks and Romans lived very comfortably without a linen shirt, he adds that 'a creditable day-labourer would be ashamed to appear in public without a linen shirt' in his time. Apart from these points, the main critic of this approach to aggregating over *sub*-capabilities comes from the assumption that any *sub*-capabilities may be overlapped with a finite amount of income.

A different solution to the problem of dependence is proposed by Chakraborti (1996). To aggregate information over *sub*-capabilities, Chakraborti establishes a correspondence between the space of capabilities and the space of utilities. In the space of utilities, people can judge the importance capabilities, separate from one another. That is, individuals may have judgements about the relative importance of the capability of being sheltered, independent of the relative importance that they concede to the capability of being employed. And they have this ability even in that case in which their capability for being sheltered directly depends on their capability for being employed. In his proposal, Chakraborti assumes that each individual has her 'own judgements about relative weights for various functionings'. From this premise, he develops an axiomatic framework to agree on the set of weights to be used in the aggregation rule across those functionings that an individual realizes.

While there is no contradiction with the capability approach in aggregating those individual values formulated in weights, there is contradiction in the use of those values to aggregate capabilities. From weighting with their weights, a measure of the value that people gives to a living style is obtained. However, this procedure does not inform us about the expansion of substantive freedoms of that living style. Let us introduce the following example.

The long history of gender inequality has shaped the preferences promoted in a society. Let us suppose that all women and men believe that education for women is not useful. And, let us also suppose that there is no possibility that women can achieve that skill of literacy. While in such a case the measure of well-being does not incorporate women's literacy. Is this a reason enough to assume that this society enables everybody to achieve the same level of well-being? We cannot indeed assume this result. A measure of deprivation should not depend on the 'mental reaction to a standard of living', to quote Sen(1983).

A basic dominance criterion is maybe the only criterion that respects the principle of dependence among *sub*-capabilities. And, at the same time, this ignores the role of individual preferences to measure well-being. Criticism of dominance criteria is raised because dominance frequently does not provide a complete ranking from which to have well-being measured. And completeness may be very relevant for social choice of public policies for the enhancement of well-being. In what follows, after introducing a representation of well-being, we discuss a basic dominance criterion to rank capability sets.

2.1. A Representation of Individual Well-Being

Sen (1985) formally defines 'capabilities' of person i, $Q_i(.)$, as a function of her entitlements over commodities, X_i, and her patterns of use of those commodities, F_i. for that case in which parameters X_i and F_i are given, capabilities of person i are displayed by the set $Q_i(X_i)$:

> $Q_i(X_i)$=[b_i / b_i= $f_i(c(x_i))$, for some $f_i \in F_i$ and for some $x_i \in X_i$], where b_i denotes a feasible combination of functionings and c(.) denotes a function converting a commodity vector into a vector of characteristics of those commodities, Sen(1985:7-9).

We want to add some structure to his definition. In order to arrive a definition of gradual capabilities from Sen's proposal, we assume the following:

> (1) General capability of an individual to live well can be seen as a family of L focal *sub*-capabilities, (∞>L>0).
>
> That is, $Q(.)=(Q^1, \ldots, Q^L)$.
>
> (2) Each *sub*-capability, Q^l, has associated a complete linear order of a set of functionings, B^l. The order is mainly interpreted as a measure of the degree in which Q^l is overlapped.
>
> (3) There exists a strictly increasing function, $\phi^l(.)$, converting functionings, $b^l \in B^l$, into real numbers. To match our results with literature, we refer to the interval [0,1] in the real line. For a further analysis on real representations, we refer to Bridges and Mehta (1995).

Thus, the definition of gradual capabilities require a family of strictly increasing functions, $\phi=(\phi^1(.), \ldots, \phi^L(.))$, with one and only one per *sub*capability. Then, gradual capabilities of a person i (given X_i and F_i), denoted $C_i(X_i)$ for that case in which parameters X_i and F_i are given, is indeed a set in $[0,1]^L$. And, this is formally defined as:

> $C_i(X_i)$=[$\phi(b_i)$ / $\phi(b_i)$= $\phi(f_i(c(x_i)))$, for some $f_i \in F_i$ and for some $x_i \in X_i$].

A long the paper, we focus on gradual capabilities defined like this.

2.2. The Definition of a Basic Dominance Criterion

Let **D** be a particular transitive, reflexive, but not necessary complete binary relationship defined over sets of elements C$\subseteq[0,1]^L$. The rule **D** is defined as:

$$\forall \text{C, C'} \subseteq [0,1]^L, \text{ C } \mathbf{D} \text{ C'} \Leftrightarrow \forall \text{c'} \in \text{C'} \ \exists \text{c} \in \text{C, such that c} \succeq \text{c'}^{1}.$$

This is indeed a basic dominance criterion defined for elements in $[0,1]^L$. Hence, C **D** C' is interpreted as the set C offers as much as well-being than the set C' does.

In section 3, we apply this rule to rank profiles of capabilities on the basis of their common capability set (that set of combinations of functionings any of which any person may realize). It is a matter of fact that this rule may not be complete. This is not in that case in which C=$\{(0'1, 0'9), (0'8, 0'5)\}$ and C'=$\{(0'2, 0'8), (0'5, 0'7)\}$, C and C' can not be compared each other.

Because a complete binary rule may be required to have distributions of capabilities ranked, we refine this rule in the following section.

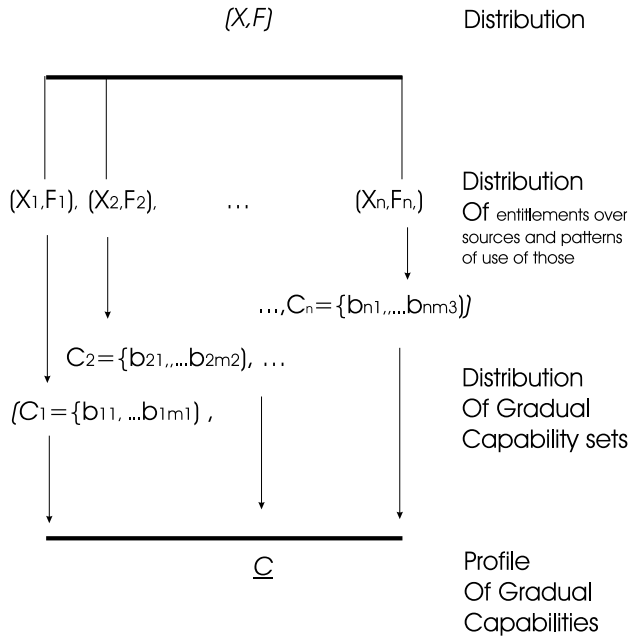
3. On Measuring Overall Well-being from distributions of capabilities

Consider a set of public policies consisting on distributing both, entitlement over sources, X, and patterns of use of commodities, F. Each public policy determines a profile of individuals' capability set, denoted $\hat{\mathbf{C}}$. We assume that all relevant information to rank these policies is compiled in their generated profile.

Before continuing, we want to illustrate the relationship between profiles of capabilities and and distributions (X,F). This relationship is pictured in Figure 1.

Figure .1.:

[1]Given c=(c^1, ...,c^L) and c'=(c'^1, ...,c'^L), c$\succeq$c' means that $c^l \geq c'^l$ $\forall$l=1,...,L. Where $\geq$ represent the order of real numbers.



where (X,F) is some distribution of both, entitlements over sources and patterns of use of commodities, C_i denotes gradual capability set of person i and b_{im_i} denotes her m_i-th available gradual combination of functionings. And, where $\hat{\mathbf{C}}$=(C_1,. . ., C_n) denotes the profile linked to distribution (X, F).

To have information from profiles aggregated in order to measure well-being provided by a public policy, we support that point in the literature which consists on arriving to a common capability set, which is denoted $C_{\langle co\rangle}$. For this aggregation, we do not assume independence among the L *sub*-capabilities.

A formal definition of common capability set, $C_{\langle co\rangle}$, is provided in what follows.

$C_{\langle co\rangle}$= [c∈$[0,1]^L$/ c$\succeq\phi(b)$ for some $\phi(b_i)\in C_i$ ∀i=1,. . .,n]

For an interpretation of c$\succeq\phi(b)$, we refer to footnote 1.

This is indeed the set of elements in $[0,1]^L$ with the particular characteristic that everybody may achieve (or at least improve) from a transformation of their available combinations of functionings into [0,1].

3.1. On ranking profiles of capability sets.

To have profiles of capabilities ranked on the basis of their common capability set, an extension of dominance aggregation rule is defined in what follows.

Given two profiles of capability sets $\hat{\mathbf{C}}$, $\hat{\mathbf{C}}$**'** $\in[0,1]^{Ln}$, with $C_{\langle co\rangle}$ and $C'_{\langle co\rangle}$ their respective common capability set. Given a reflexive, transitive but not necessary complete binary relation $\mathbf{R}$ defined over elements in $[0,1]^{Ln}$, this relation is defined as follows:

> $\hat{\mathbf{C}}$ **R** $\hat{\mathbf{C}}$**'** $\Leftrightarrow C_{\langle co\rangle}$ $\mathbf{D}C'_{\langle co\rangle}$, where $\mathbf{D}$ is the binary relation formerly defined over $[0,1]^{L}$.

C R C' is read as *profile **C** offers as much as well-being* for all individuals in the society than profile C'.

Indeed, this extension of dominance aggregation rule does not provide a complete ranking of profiles in $[0,1]^{Ln}$. Alternatively, Herrero, et. at (1997), and literature quoted there, provide an axiomatic framework to have a profile of capabilities ranked on the basis of common sets. They characterize three complete binary rules ('leximin opportunity relation', 'utilitarianism opportunity relativism' and 'common opportunity relation'). Those rules do not respect however the principle of dependence among sub-capabilities.

Given that dominance criterion seems to be the one which respects dependence of capabilities, we refine it to have and order of profiles of capabilities (a complete, transitive and reflexive binary rule).

3.2. On an Order for Profiles of Capability Sets

Let introduce some added notation to arrive to a complete ranking.

> P(b) denotes the ratio of the number of people who has in their gradual capabilities at least an element $c_i \in C_i$ such that $c_i \succeq \phi(b)$ (for an interpretation of $c_i \succeq \phi(b)$, we refer to footnote 1), and this number of people of the total of people in the society. The ratio P(b) can be read as the probability of any individual of having b combination of functionings in her capability set.

> $C_{\langle j\rangle}$ denotes the set of combinations of functionings, b, that P(b)=j.

Each profile of capability set has associated an order of capabilities , from $C_{\langle 1\rangle}$ to $C_{\langle 0\rangle}$, where $C_{\langle 1\rangle} \subseteq \ldots \subseteq C_{\langle 0\rangle}$. Let note that $C_{\langle 1\rangle} \equiv C_{\langle co\rangle}$.

An option to have profiles completely ranked consist of making use of the lexicographic order, R_{LO}. An extension of it is defined such as follows:

$\forall \hat{\mathbf{C}}, \hat{\mathbf{C}}' \in [0,1]^{Ln}$,
$\hat{\mathbf{C}} R_{LO} \hat{\mathbf{C}}' \Leftrightarrow \exists$a$\in$[0,1] such that $C_{\langle a\rangle}$ **D** $C'_{\langle a\rangle}$ and $C'_{\langle b\rangle}$ $\neg$**D** $C_{\langle b\rangle}$ $\forall$ b $\geq$ a.

The lexicographic order defined with information from both, space of capabilities and space of probabilities, defines and order over elements in $[0,1]^{Ln}$. And, the principle of dependence among capabilities is respected.

4. Conclusion

To have the overall well-being from distributions of capabilities measured, a complete order of profiles of capability sets may be wanted. Since we are keen of researching into more than one *sub*-capability to arrive at a measure of well-being, an aggregation rule should be specified. Most of the common aggregation rules assume independence among dimensions. In the case of capabilities, there is no chance to assume this property. Capabilities depends one of the others to provide well-being to individuals. The lexicographic order defined with information from both, space of capabilities and space of probabilities, defines and order over elements in $[0,1]^{Ln}$. And, the principle of dependence among capabilities is respected.

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