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Endogenizing Sen's Capabilities:
An Adaptive Dynamic Analysis

Antonio D'Agata
D.A.P.P.S.I.
Via Vittorio Emanuele, 8
95131 Catania
Italy
Tel. 095 746 21 11
Fax 095 746 41 93
Email adagata@unict.it

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Abstract.

One of the main features of the current state of the literature on capabilities as introduced by Sen (see e.g. Sen (1985)) is that the characteristic and the utilization functions are conceived as exogenously given. Sen himself has recently stated that capabilities could be considered as a more general concept of human capital (Sen, 1997). Hence, learning in extracting characteristics from goods and in utilizing characteristics should be accounted for and this, in turn, implies that characteristics and utilization functions can no longer be considered as given. Therefore, Sen's capacity approach calls for a reconsideration within a dynamic context.

The aim of this paper is to develop a quite general formal model in which the utilization functions are determined endogenously through an adaptive learning process. The dynamic model we adopt is a simple multi valued dynamic model (see e.g. Day and Kennedy (1970), Aubin (1997)). Because of this feature, the model, although it is not stochastic, may exhibit non deterministic dynamics. Among other results, we provide rigorous results concerning the dynamics of the sequence of states generated by the adaptive process and existence of an equilibrium configuration in utilization functions. By means of numerical examples we also show that an individual's capabilities may evolve in such a way that the individual may be locked in a trajectory of utilization of characteristics which leads him/her to a sub-optimal configuration. Finally, based upon our findings we draw some conclusion on the Sen's capability-based theory of distributive justice.

J.E.L. Classification Codes: C62, D11, D63.

Keywords : Capabilities, multivalued dynamics, adaptive dynamics.

> “Thus with ignorance as cause there are the aggregates,
> with the aggregates as cause there is consciousness…
> with rebirth as cause there is old age and death.
> Even so is the origin of this whole mass of pain”
> *Samyutta-nikāya*, ii, 10.

1. Introduction

Since the seminal work of Sen (1979), in which capabilities are introduced as a basis upon which equality should be evaluated, the concept of capabilities has been object of wide study by the profession, both at theoretical and at empirical level (see e.g. Nussbaum and Sen (1993), Sen (1997a) and references therein, Pattanaik and Xu (2000), Gekker (2001))). Capabilities are determined through utilization functions (Sen, 1985, Chapter 2), which associate a vector of states (functioning) to each vector of characteristics. One of the main features of the current state of the literature on capabilities is that the utilization functions are conceived as exogenously given.

A major implication of Sen’s approach is that any problem concerning equality should preliminarily answer to the question the right choice of variables (the *focal variables*) according to which interpersonal inequality has to be evaluated. Once the focal variables are chosen, then the allocation of resources should be allocated in such a way to ensure the equality of capabilities in the focal variable space (Sen, 1992, Chapter 1). However, Sen emphasises that equality in a variable space may not necessarily ensure the equality in another variable space, because individuals differ in the way in which they are able to convert characteristics into functionings; as a matter of fact, equal allocation of resources (i.e. equality in the space of goods) can well imply, for example, a different distribution in agents’ well-being (i.e. inequality in the space of well-being). According to Sen, this problem cannot arise if agents are exactly the same:

> “Had all people been exactly similar, equality in one space (e.g. incomes) would tend to be congruent with equalities in others (e.g. health, well-being. happiness)” (Sen 1992, p. 20)

Hence, according to Sen in pursuing egalitarian policies one must be careful not to replace the equality of well-being with the equality of resources used to attain a certain level of weel-being.

Sen has recently stated that capabilities can be considered as a more general concept of human capital (Sen, 1997b). If this interpretation is accepted, then learning in extracting characteristics from goods should be accounted for and this, in turn, implies that utilization functions can no longer be considered as given. As the current literature considers utilization functions as exogenously given, Sen's capacity approach calls for a reconsideration within a dynamic context

The aim of this paper is to develop a very general formal model in which the utilization functions are determined endogenously through an adaptive learning process. After sections 2 and 3 where Sen's formalization of his approach is presented and the need for a dynamic analysi is justified, in section 4 the dynamic model is developed. It is a simplified version of the recursive model developed by Day and Kennedy (1970) and a discrete-time version of the evolutionary approach proposed by Aubin (1991 and 1997). The interesting fact about this approach is that it is non-stochastic although the dynamics it generates is not necessarily deterministic. Because of the last properties, our model allow for path-dependent phenomena (David, 2000). We provide rigorous results concerning the dynamic properties of the sequence of states generated by the adaptive process and the existence of an equilibrium configuration in utilization functions, i.e. a configuration in which learning in improving utilization functions is exhausted. In sections 5, by means of numerical examples we point out that once the learning process is introduced, then the evolution of capabilities may no longer be separated by the evaluative elements (Sen, 1985). More specifically, it is shown that exactly equal individuals facing the same environment can experience an evolution of utilization functions such that they end up with different functionings, some of the individuals being locked in a trajectory of utilization functions that leads him/her to a sub-optimal functioning configuration. This result seems to weaken considerably Sen's view, already discussed, according to which differences in different spaces are due mainly to differences among individuals, and complicates further any theory of distributive justice. This problem will be dealt with in sections 6 and 7.

2. Capabilities and functionings: a formal description

Chapter 2 of 1985 Sen's book, *Commodities and Capabilities*, provides a formal account of his view of capabilities, functionings and well-being. In this chapter, Sen first emphasises the importance of characteristics of commodities – i.e. the “various desirable properties of the commodities in questions” (Sen, 1985, p.9) - in satisfying individual’s needs. Then, he emphasises again that the ownership of commodities cannot be used to measure the well-being of people, it being determined instead by “what the person succeeds in *doin*g with the commodities and characteristics at his or her command” (Sen, 1985, p. 10). Eventually, he introduces his ideas of functionings and capabilities in formal way. Consider an individual with a *n*-dimensional commodity space, R^n_+ and with an associated characteristic space C. Let x be a generic vector of commodities owned by the agent and c: $\Re^n_+ \rightarrow C$ the characteristic function associating to each vector bundle in $\Re^n_+$ a vector of characteristics in C. The agent is assumed to be able to extract characteristics of bundle x, $c(x)$, according to a “utilization function” f: $C \rightarrow S$, where S is the set of functionings of the individual. The function h: $S \rightarrow \Re$ is the “happiness function” and v: $S \rightarrow \Re$ is the “valuation function” of the individual (*Ibidem*.). Finally, Sen assumes that each individual is endowed with a multiplicity of utilization functions, F and that he/she can choose any vector x in a subset X of $\Re^n_+$. Hence, $c(x)$ is the vector of characteristics associated to bundle x, $f(c(x))$ is the functioning the individual can attain by owning vector x and by using the utilization function f; finally, $\{s \in S | \ s = f(c(x)), x \in X\}$ is the capacity set of the agent. The real number $h(f(c(x)))$ measures his/her happiness while $v(f(c(x)))$ measures the valuation of agent’s life.

3. Human capabilities and human capital

At the end of Chapter 1 of *Commodities and Capabilities* and following the lancastrian approach to consumer behaviour according to which the household is a “small factory” (Lancaster (1966a), Becker (1976)), Sen recognises that, although with qualifications, functionings can be interpreted as the output of the household (Sen, 1985, p. 15). It follows that characteristic and utilization functions can be interpreted as technologies. Sen emphasizes that this interpretation cannot be accepted *in toto* for three reasons: first, as the output (functionings) “are features of the state of existence of a person, and not detached objects…”(*Ibidem*.). Second, because “questions suggested by the analogy may also not be always especially appropriate (e.g. what is ‘the time required to produce a unit of commodity’)” (Sen, 1982, p. 16). Third, because many functionings are produced outside the household (*Ibidem*.).

However, in a short article in 1997, Sen comes back to the interpretation of utilization functions as technology by analysing the relationship between human capital and human capabilities. Starting from the fact that each person has the ability to do certain things that he/she considers valuable, Sen emphasises that the dichotomy between human capital and human capabilities depends upon the fact that there are two different reasons for valuation: direct, in the sense that the functioning involved improves directly agent’s life, or indirectly, in the sense that by means of it the agent can contribute further to production (and, therefore, command a price). According to Sen, the human capital approach “concentrates on agency of the human beings –thorough skill and knowledge as well as effort – in augmenting production possibilities. The latter [i.e. the human capability approach] focuses on the ability of human beings to lead lives they have reasons to value and to enhance the substantive choices they have” (Sen, 1997b, p. 1959). In fact, unlike the human capital approach which traditionally values indirectly human qualities like capital goods, the human capability approach includes and values also those human qualities providing a direct improvement in life, even if they are not marketable. [1]

If we adopt this wider view of capabilities, then we face immediately the fact, left out until now by the literature, that, like human capital, human capability is a dynamic notion. Economists, particularly in recent years, have extensively studied the dynamics of human capital and, in particular, its relation to economic growth (see e.g. Lucas, 1988, Romer, 1986). According to this stream of research, productivity of labour increases either for facts which are beyond the deliberate decision of agents, like learning-by-doing, or because it is the outcome of an investment, usually by means of education. In both cases, the improvements in the quality of labour yields an increase in the market value of the labour itself. Usually, the literature studies the intertemporal allocation of labour in productive (as work

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[1] Sen’s view is close to the view of Hobson (1914), Mitchell (1912) and Scitovsky (1976) concerning consumers’ skills. Notice that, according to Scitovsky, the non marketability of consumption skills is at the origin of the lack of interest of people of the United States for culture (Scitovsky, 1976, Chapter 11).

and investment in human capital) and unproductive activities, and their effects on economic growth.

More recently, orthodox and heterodox economists have shown an active interest also in deepening their understanding of consumer behaviour by using the lancastrian approach of consumer technology. One of the main point emphasised by this literature is the importance of the growth over time of the knowledge of the consumer's technology (see e.g. Romer (1996), Bianchi (1999), Nooteboom (2000, Ch. 10). See also Lancaster (1966b)).

If we extend this view to the broader concept of human capabilities then we can conclude that capabilities as well should be analysed in a dynamic context, as individual can change, deliberately or not, their utilization functions (or the set of utilization functions) or their characteristic functions over time.[2] This step seems to be a particularly important aspect to consider if, interpreting the capability approach as a framework of thoughts, we intend to apply this approach for policy evaluation or inequality measurement in affluent societies (see e.g. Robeyns 2001, p. 11). In these societies, in fact, the goods and services available can be so complex that a substantial expertise may be required to extracts all the features of them, and such expertise can be obtained only through a period of learning.

In the next section, we shall develop an abstract adaptive model in which the set of available utilization functions growths over time and agents choose at each period the "best" utilization function, according to an objective function (which can be either the utility function or the valuation unction). For the sake of simplicity, we assume that there are no improvements in the knowledge of characteristic functions, although our analysis can easily incorporate this case. We shall provide sufficient conditions for the dynamics generated by the adaptive process to be convergent, and that the limit point is an equilibrium configuration. In section 5, by means of numerical examples, we also shall show that individuals can be trapped into sub-optimal configurations in using utilization functions. Finally, in sections 6 and 7 we shall draw some implications of our findings to the theory of distributive justice.

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[2] Sen himself has considered the utilization functions as depending upon several time-dependent parameters like age, health, social conditions, education (Sen, 1983, Chapter 12). However, to the best of our knowledge no explicit dynamics analysis has been developed from this.

4. An adaptive model of evolution of functionings

We shall introduce dynamics in the model described in Section 2 as follows: Time is a discrete variable: 0, 1, 2,….. At time 0 an agent is endowed with a given bundle of commodity $x_0 \in X$, with *one* characteristic function $c : X \rightarrow C$ and with a given set of utilization functions, $F_0 \subset F$, where $f \in F$ is defined in C with images in S. As already said, we assume that no change occurs to the (singleton) set of characteristic functions $\{c\}$ over time. Moreover, we shall assume that a real valued objective function ω is defined on S. This function could be interpreted indifferently as a valuation function v or an happiness function u. Setting $c_0 = c(x_0)$, at time 0 the agent has a capacity set $K_0 = \{s \in S | \ s = f(c_0), f \in F_0\}$. Notice that, for the sake of simplicity, in the above definition of capacity set K_0 we do not allow for "free disposal" (i.e. the possibility of choosing bundles $x \leq x_0$). The functioning correspondence $\Sigma : F \rightarrow S$ is defined as follows: $\Sigma(f) = \{s \in S \ | s = (f(c_0))\}$.

The following adaptive process defines the behaviour of the consumer:

Adaptive Process: At the beginning of time 0 the agent will choose the utilization function f_0 which maximizes the objective function ω on the feasible set of utilization functions F_0 i.e. he/she will choose any element of the set: $B\ (F_0) = \{f \in F_0 \ | \ \omega(f(c_0)) \geq \omega(g(c_0)), g \in F_0\}$, and the value $\omega_0 = \omega(f_0)$ is attained. During period 0, once the utilization function f_0 has been chosen, the agent will carry out "experiments around" f_0 in such a way that, at time 1, he/she has available a new set of utilization functions "discovered" at time 0, $F(f_0) \subset F$ and, therefore, a new set of potentially available functionings: $\Sigma(F(f_0))$. At time 1 a utilization function f_1 will be chosen in the set of optimal utilization functions $B(F(f_0)) = \{f \in F(f_0) \ | \ \omega(f(c_0)) \geq \omega(g(c_0)), g \in F(f_0)\}$, and the value $\omega_1 = \omega(f_1)$ is attained. And so on.

Thus, for every initial set F_0, the adaptive process just described defines a sequence of utilization functions in F, $\{f_t\}$, of functionings in S, $\{s_t\}$, and of values in $\Re$ $\{\omega_t\}$ where $f_t \in B(F(f_{t-1}))$, $s_t = f_t(c_0)$ and $\omega_t = \omega(f_t(c_0))$, and where $f_0 \in B(F_0)$.

From this description of the adaptive process the following definition of equilibrium is natural:

Definition 1. A configuration $f^* \in F$ is an *adaptive equilibrium in functionings* (*AEF*) if $f^* \in B(F(f^*))$.

An *AEF* is a utilization function $f^* \in F$ such that f^* maximizes the value of the objective function ω over the set of utilization functions which are available when f^* is chosen.

In the next two sections, we shall provide answers to the ensuing questions:

1. Is there any AEF?
2. Does the sequence of states generated by the adaptive process converge towards an AEF?
3. Does the sequence of states generated by the adaptive process converge towards only *one* AEF?
4. If there is more than one AEF, does the sequence converges towards the AEF which is the best among all possible equilibria?

More specifically, in this section we shall answer to questions 1-3, question 4 will be answered in the next section.

Assumption 1. Sets F and $S' = \{s \in S \mid s = f(c_0), f \in F\}$ are compact. Set F is also a metric space.

Assumption 2. Set F_0 is closed; moreover, for every $f \in F$, set $F(f)$ is closed.

Assumption 3. The objective function ω is continuous over S.

The first part of the following result answers positively to question 1, while the second part answers positively to question 2 above. In this result we exploit the fact that, by Assumptions 1 and 2, sets F and S' are compact and, therefore any sequence in them *has* an accumulation point.

Theorem 1. Under Assumptions 1-3 the sequence $\{f_t\} \subset F$ generated by the Adaptive Process from any set $F_0 \subset F$ has at least a converging subsequence; moreover, every limit point of this sequence is an AEF.

Proof. Set: $\Omega = \{f \in F \mid f \in B(F(f))\}$. Clearly, every $f \in \Omega$ is an AEF. Notice that the sequence of utilization functions $\{f_t\}$ generated by the adaptive process is obtained by the algorithm defined by the correspondence $B(F(f_t))$ with set Ω as solution set (see Zangwill, 1969, Chapter 4). By Assumption 1, all points f_t belong to the compact set F, therefore, the sequence has at least one converging subsequence.

If $f \notin \Omega$, then for every $g \in B(f)$, $\omega(g(c_0)) > \omega(f(c_0))$, while if $f \in \Omega$, them for every $g \in B(F(f))$, $\omega(g(c_0)) \geq \omega(f(c_0))$. Finally, by Assumptions 1, 2 and 3 and by Berge

Theorem (Berge, 1963), the correspondence $B(\cdot)$ is upper hemi-continuous. Therefore, by Zangwill's Convergence Theorem A (Zangwill, 1969, p. 91), the limit f^* of any convergent subsequence of $\{f_t\}$ belongs to Ω. From what has been said at the beginning of this proof, f^* is an AEF. ■

As said before, Proposition 1 is a quite general result, since it shows not only that, under the stated assumptions, the economy converges to the AEFs, but it also ensures, thanks to a standard compactness argument, the *existence* of a AEF.

As far as question 3 is concerned, a shortcoming of Proposition 1 is that if the Adaptive Process generates an infinite sequence of chosen utilization functions $\{f_t\}$, this sequence may converge to two or more different AEFs. This behaviour is clearly not satisfactory from the empirical point of view as it might imply not coherent behaviour of individuals over time. A straightforward way to exclude this possibility is to assume that there is a unique AEF. In this case, in fact, an easy argument shows that the whole convergent sequence generated by the Adaptive Process converges to the AEF(see e.g. Bazaraa, Sherali, Shetty (1993), p.250). The remaining part of this section is devoted to provide two alternative sufficient, more palatable from the economic point of view, conditions ensuring that the whole (infinite) sequence converges to a single AEF.

Assumption 4: There exists a family of disjoint compact neighbourhood of the AEFs, say $\mathcal{N}$, which satisfies the following conditions: (i) each element of the family contains at most one AEF and, (ii) if f^* and f^{**} are two different AEFs with $f^* \in N^*$ and $f^{**} \in N^{**}$ where $N^*, N^{**} \in \mathcal{N}$, then $F(f^*) \subset N^*$ and $F(f^{**}) \subset N^{**}$.

Assumption 4 ensures that $f^* \notin F(f^{**})$ and $f^{**} \notin F(f^*)$; that is, this assumption ensures each AEF cannot be reached from any other AEF. The following proposition states that Assumptions 1, 2, 3 and 4 are sufficient for generating nice dynamics as described above.

Proposition 2: Under Assumptions 1, 2, 3 and 4 the whole (infinite) sequence generated by the Adaptive Process converges to only one AEF.

Proof. First we show that if $\{f_t\}$ is a sequence of utilization functions generated by the AP, then $d(f_t, f_{t+1}) \rightarrow 0$ as $t \rightarrow \infty$, where d is any metric defined on the set F. Suppose not. Then, there exists a sub-sequence $\{f_t\}$ such that $d(f_t, f_{t+1}) \rightarrow \beta > 0$ as $i \rightarrow \infty$. We may assume that $\{f_t\}$ converges to f^* and that $\{f_{t+1}\}$ converges to f^{**}.

Clearly, $d(f^*, f^{**}) \geq \beta$. By Proposition 1, f^* and f^{**} are AEFs; moreover, $f_{t+1} \in F(f_t)$, then by Assumption 3, $f^{**} \in F(f^*)$ But this contradicts Assumption 4.

Suppose now that the assertion of Proposition 2 is not true. Therefore, if $\{f_t\}$ is the sequence generated by the Adaptive Process, then there exist (at least) two subsequences, say $\{f_{t'}\}$ and $\{f_{t''}\}$ converging to f' and f'', respectively. By Proposition 1, every accumulation point of sequence $\{f_t\}$ is an AEF; therefore, by Assumption 4, it is possible to take two positive numbers ε' and ε'' such that points f' and f'' are the only accumulation points in the balls of radius ε' and ε'' around f and f', respectively, i.e. $B_{\varepsilon'}(f')$ and $B_{\varepsilon''}(f'')$, where $B_{\varepsilon'}(f') \subset N(f')$ and $B_{\varepsilon''}(f'') \subset N(f'')$. Choose a positive number Z in such a way that $d(f_z, f_{z+1}) < \varepsilon'/3$ for $z \geq Z$ (That such a number exists follows from the result at the beginning of this proof). However, (f') is an accumulation point of sequence $\{f_t\}$, therefore for infinitely many indices s one has that $d(f_s, f') < \varepsilon'/3$. On the other hand, (f'') is another accumulation point of $\{f_t\}$, hence, by the fact that $(f'') \notin B_{\varepsilon'}(f')$, there must exists infinitely many indices q such that $d(f_q, f') \geq 2\varepsilon'/3$. From the way in which Z has been defined, it follows that there exist infinitely many indices $m \geq Z$ such that $(\varepsilon'/3) \leq d(f_m, f') \leq (2\varepsilon'/3)$. This implies that there exists an accumulation point in the set $\{f \in F \mid (\varepsilon'/3) \leq d(f, f') \leq (2\varepsilon'/3)\} \subset N(f')$, which contradicts Assumption 4.

■

Actually, Assumption 4 imposes conditions on the "size" of the sets of utilization functions learnt when the agent is at AEFs. However, especially in the case of learning by doing, it is natural to assume that learning by doing is bounded, i.e. that the incremental discovery of new utilization functions tends to zero as time passes (see, e.g.,Young (1993, especially footnote 3). The boundedness of learning by doing in our approach can be formalised by assuming that the "size" of set $F(f_t)$ tends to zero as time flows.

Assumption 5. $diam(F(f_t)) \to 0$ as $t \to \infty$ where $diam(F(f_t) = inf\{n \mid B_n \subset F(f_t)\}$ and B_n denotes a ball of radius n in $F(f_t)$.

Assumption 5 ensures the following result, whose proof is trivial.

Proposition 3. Under Assumptions 1, 2, 3 and 5 the (infinite) sequence of techniques generated by the Adaptive Process converges to a AEF.

5. Dynamic inefficiency: two examples

In this section we provide two examples which not only make it clear the logic of the abstract model introduced in the preceding section in a clearer way, but which also show that the evolution of functionings may exhibit lock in phenomena with path dependent inefficiency because eventually an individual ends up with lower functionings than the maximum attainable simply because he/she us locked in a “wrong” trajectory.

Consider an economy with only one durable good, having two characteristics and two variables describing the functionings; i.e. $C = S = \Re^2$. There are two individuals, indicated by 1 and 2, each having one unit of the good once and for all,[3] the same characteristic function defined by the function $c(x) = (x, x)$, and an objective function defined as follows:

$$\omega(x_1, x_2) = \begin{cases} \min(x_1, x_2/3) \text{ if } x_1 \leq x_2 \\ \min(x_1/3, x_2) \text{ if } x_1 > x_2, \end{cases}$$

where (x_1, x_2) represents the functionings attained by the agent. Both the characteristic function and the objective function are assumed to be constant over time. We assume moreover, that the utilization function evolves over time, and that the generic utilization function available at time t is indicated by f_t. For the sake of simplicity, we assume that the initial utilization function and the exploration of new utilization functions occur within the space of linear mapping from $\Re^2$ into $\Re^2$. The generic element of this set is indicated by a matrix $f_t = \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} = \begin{bmatrix} a_{11t} & a_{12t} \\ a_{21t} & a_{22t} \end{bmatrix}$, while matrix $\overline{f_t} = \begin{bmatrix} \overline{f}_{1t} \\ \overline{f}_{2t} \end{bmatrix} = \begin{bmatrix} \overline{a}_{11t} & \overline{a}_{12t} \\ \overline{a}_{21t} & \overline{a}_{22t} \end{bmatrix}$ indicates the utilization function chosen at time t.

We assume that F_0 is singleton and that the set of utilization functions and its dynamics are given by the following rules:

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[3] Alternatively, we can assume that one unit of perishable good is given to the agents at each period.

1. $F = \{ \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} = \begin{bmatrix} a_{11t} & a_{12t} \\ a_{21t} & a_{22t} \end{bmatrix} \in \Re_{+}^{2\times 2} \mid a_{ijt} = 0, i, j = 0\ 1, 2, i \neq j, \ \ 0 \leq a_{11t} \leq 15, 0$ $\leq a_{22t} \leq 5$ for every $t\}$,
2. $f_0 = \begin{bmatrix} f_{10} \\ f_{20} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$
3. $F(\overline{f}_{t\text{-}1}) = \{ \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} = \begin{bmatrix} a_{11t} & a_{12t} \\ a_{21t} & a_{22t} \end{bmatrix} \in \Re_{+}^{2\times 2} \mid a_{ijt} = 0, i, j = 1, 2, i \neq j, \ a_{11\mathrm{t}} = \max (1, \overline{a}_{11t\text{-}1}) + \alpha(\max (1, \overline{a}_{11t\text{-}1}) + 1 - 2\lambda), a_{22\mathrm{t}} = \max (1, \overline{a}_{22t\text{-}1}) + \alpha(\max (1, \overline{a}_{22t-1}) + 2\lambda - 1), \alpha, \lambda \in [0, 1]\}$

Condition 1 says that the set of linear mappings among which all possible utilization functions are explored and chosen is a subset of the set of 2x2 diagonal matrices. Condition 2 defines the sole utilization function available at time 0 (i.e. the identity map). Condition 3 provides the rule of exploration of the space of the utilization functions.

Figure 1 illustrates the dynamics of the model. Curves I_1, I_2 and I_3 represent the indifference curves associated with levels 1, 2 and 3 of the value of the objective function, respectively. By condition 1, the rectangle 0-5-M-15 represents the set of all states potentially attainable by the individuals.

At time 0, the functioning vector available to each agent is point A (i.e. vector (1,1)). By condition 3, the triangle ABC is the set of states that can be reached at time 1, due to the improved knowledge in the utilization functions. States B and C are indifferent to agents, so it is possible that agent 1 chooses states B and agent 2 states C. Given the information at time 0, the choice of state B by 1 and of state C by 2 can be due just by chance. By the rule $F(\cdot)$, the triangle BED is the set of states which agent 1 can reach at time 2, while the triangle CFG is the set of states that agent 2 can reach at time 2. Clearly, agent 1 stops at point D and enjoys state (3, 5), while agent 2 chooses point H, and exploration of its state space will continue: at time 3 he will choose point L and at time 4 eventually point M = (15, 5).

From this example we can conclude that identical individuals facing the same (deterministic) environment can end up with different utilization functions (and, consequently, with different capabilities). Moreover, the difference is due to differences in the trajectories of chosen utilization functions over time, which in turn cannot be explained in terms of different performance between individual but should be considered the result of sheer luck. This outcome, moreover, is independent of the adaptive behaviour concerning preferences that characterises the “tamed widow” case, because preferences here are given. However, in our analysis preferences still play a crucial role in determining the evolution of capabilities as they determine the choice of the utilization function chosen at each period, which in turns affects the exploration of the utilization function space.

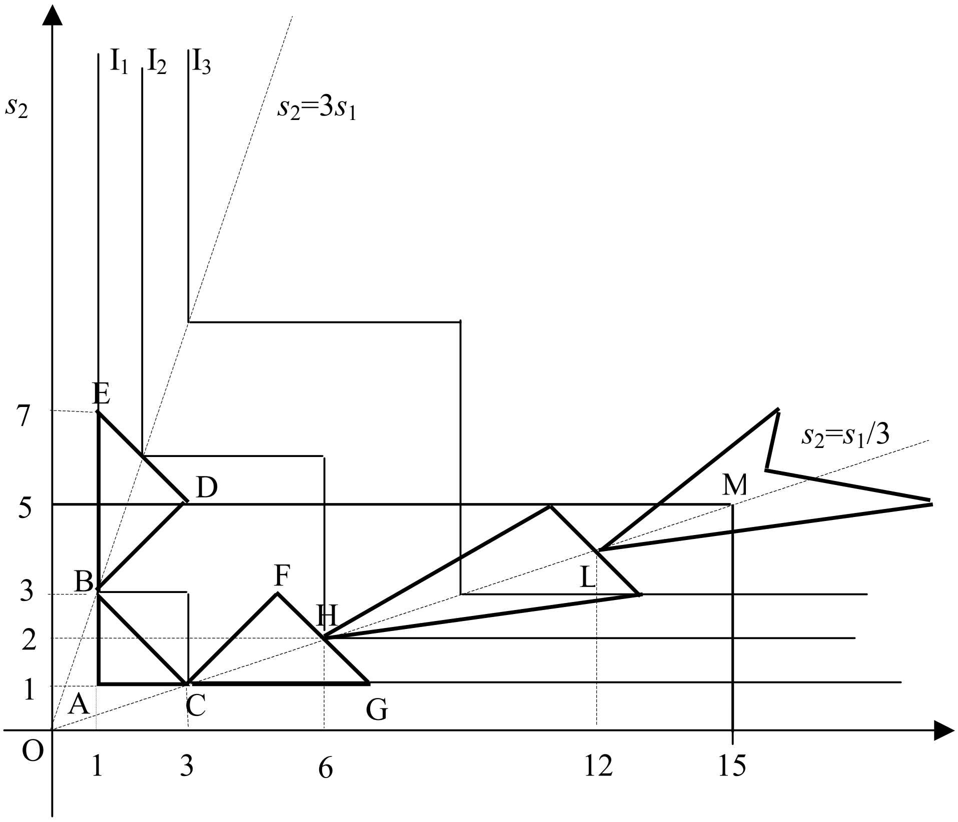

Figure 1: Dynamics of capabilities generated by growth of knowledge in utilization functions.

A further development of the previous example allows to obtain the same result in case in which, in case of ties, the agent carries out a trial-and error behaviour over all indifferent choices. Suppose now that the correspondence F defining the available utilization functions next period is:

$$F(\bar{f}_{t\text{-}1}) = \{ \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} = \begin{bmatrix} a_{11t} & a_{12t} \\ a_{21t} & a_{22t} \end{bmatrix} \in \Re_{+}^{2\times 2} \mid a_{ijt} = 0,\ i, j = 1, 2,\ i \neq j,\ a_{11\mathrm{t}} = [\max(1, \bar{a}_{11t\text{-}1}) + \alpha \min\left(\frac{\bar{a}_{22t-1}}{\bar{a}_{11t-1}} \right) (\max(1, \bar{a}_{11t\text{-}1}) + 1 - 2\lambda)],\ a_{22\mathrm{t}} = [\max(1, \bar{a}_{22t\text{-}1}) + \alpha \min\left(\frac{\bar{a}_{22t-1}}{\bar{a}_{11t-1}} \right) (\max(1, \bar{a}_{22t-1}) + 2\lambda - 1)],\ \alpha, \lambda \in [0, 1]\}.$$

In this case, at time 0 any

agent will choose *both* point A *and* point B (see Figure 2), however at time 1, as it is immediate to check from Figure 2, any agent will find it profitable to choose point D and to stop the search process. However, if he/she had chosen point he would have moved towards point M (represented in Figure 1).

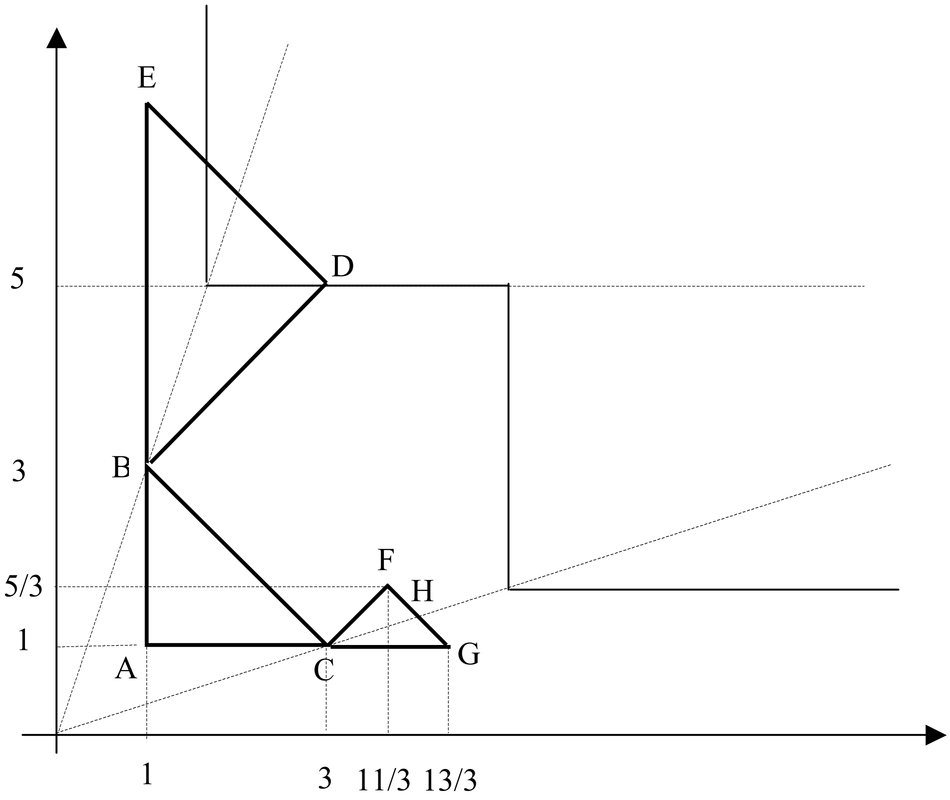

Figure 2: The case in which a trial-and-error process is activated in each indifferent optimal utilization function.

The above examples show that exactly identical individuals facing the same environment can end up with different functionings. This result draws some doubt on Sen's opinion, quoted in section 1, according to which if agents were the same, then the equality in one space should ensures the equality also in any other space. By contrast, the two preceding examples show that this is not true: for similar agents, it is possible that the equality in the goods (and characteristics) space does not ensure the equality in the space of functionings. As diversity of human beings is the main cause, according to Sen, of the failure of approaches different from the capability-based one to ensure equality, the preceding results point out that the diversity we have to take into account is more radical than the one considered so far by the literature, it being due not only to "objective" facts like sex, gender and health, but also to more subtle and subjective facts, like cognitive capacities and learning.

This fact arises new problems in the theory of distributive justice, and this will be considered in the next two sections.

5. Endogenous functionings, equality and responsibility

One of the main thrusts of Sen's approach is that it provides an alternative approach to evaluate the fairness of resources distribution. More specifically, it is reasonable to believe that Sen would equalize capacities (see e.g. Sen (1992); see also Roemer, (1993)). Within this problem, it has been recognised that equalization should be ensured within the limit of responsibility of each agent for choices affecting his/her endowment (see, for example, Dworkin (1981a, 1981b), Arneson (1989, 1990), Cohen (1990)). Arneson, for example, holds the position that what should be equalized is the opportunity of well being, in the sense that there must be a point in time at which people can have access to the same bundle of resources, and possible successive inequalities should be the outcome of choices under their control. It seems reasonable to think that Sen follows this approach, not allowing insurance for low level of capabilities which are a person's own responsibility (Sen (1992, Chapter 9)). One of the main features of this stream of literature is that it assumes that all relevant variables in which responsibility should be determined are in some way observable (in Dworkin, for example, the probability of becoming handicapped, in Arenson, the possibility to calculate the decision tree).

The example presented in the previous section suggests the possibility that two persons who are exactly the same and who face exactly the same (deterministic) environment can turn out with different levels of well being, and this not necessarily being the outcome of deliberate choices (for example, of effort) but rather of pure chance (choosing point A rather than point B). More in general, we can draw the conclusion that a population of identical individuals facing the same (deterministic) environment may show a *distribution* in the achievement of well being, although there are no differences in efforts. Therefore, following the responsibility approach indicated above, in this case the difference in capabilities should be compensated, because it cannot be attributed to the individuals' responsibility.

However, the problem is whether an egalitarian planner can obtain enough information in order to distinguish between difference in capabilities due to the particular type of evolution of utilization functions (with multiplicity of optimal utilization functions at a point in time) or because of deliberate choices or even by dissimulation. If she is able to do so, then she should compensate differences in final functionings. It is hard to believe that the planner is able to know the dynamics of the knowledge of each individual. If this is the case, then there is no way to discriminate between differences originated by lack of responsibility and difference due to pure luck. Hence any compensation schema based upon responsibility might be doubtful.[4]

It is worth noticing that the incentive problem has been deliberately overlooked by Sen, who, being interested in equalizing basic capabilities, deals mainly with non manipulable differences among individuals (sex, age, etc.) rather then with differences which could be the outcome of deliberate choices (Sen (1992, p. 141 ff)). While the analysis of basic capabilities justifies the use of non manipulable differences, in affluent societies, inequalities should be analysed in terms of complex capabilities, for which the evolution of utilization functions can be crucial. In these cases, our findings could be of some interest.

6. Cheap preferences or efficient functionings?

One of the main point emphasized by the egalitarian literature is the distinction between "cheap" and "expensive" tastes (see Sen 1985b, p. 196 and references therein). It is well established by the follower of the utility approach to equality, that, apart some specific case, no compensation should be given for the latter, as tastes are considered under the responsibility of each person (see e.g. Dworkin (1981)). The model developed in the preceding sections is quite general and can be applied in dealing with this kind of problem, just interpreting the objective function as an utility function. The first example in section 4 shows that although agents 1

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[4] A similar problem has been pointed out, in a different context, by Balestrino (1994). See also Sen's answer (Sen (1994).

and 2 have the same endowment, the same preferences and face the same environment, they end up with different levels of utility. The question naturally arises: Should the planner, who does not know the preferences of the agents, conclude that agent 1 has expensive tastes and agent 2 cheap tastes, because although endowed with the same good bundle, the final satisfaction of agent 1 is lower than that one of agent 2? A reasonable answer seems to be: no. So, we should compensate agent 1 with respect to agent 2. However, the problem, already pointed out in the preceding section, arises concerning the possibility for the planner to distinguish between the case in which “expensive tastes” are due to the structure of preferences and the case in which they are the outcome of an inefficient development of human capabilities. The possibility to discriminate further between these two cases seems to be crucial as in the case of “expensive tastes” due to the structure of preferences, as said above, no compensation should be ensured; by contrast, the case in which “expensive tastes” are due to inefficient development of human capabilities compensation must be ensured only in case in which the individual cannot be considered responsible for such an inefficiency.

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