

## **On the Arbitrariness and Robustness of Multi-Dimensional Poverty Rankings.**

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### *Abstract.*

It is often argued that multi-dimensional measures of well-being and poverty are *ad hoc* and not robust to changes in the selection of weights used. In this paper, it is argued that the extent of arbitrariness and the range of issues relating to robustness has been underestimated in this context. Several issues relating to both the identification of the poor and the use of dimension-specific data are distinguished. These issues are then investigated in the context of the inter-provincial ranking of poverty in South Africa. It turns out that this ranking is fairly robust, and some important policy-relevant results in the literature about the distinction between ‘income’/‘expenditure’ and ‘human’ poverty for the South African context are reinforced rather than undermined by checking for robustness.

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## **On the Arbitrariness and Robustness of Multi-Dimensional Poverty Rankings.**

### *0. Introduction.*

There is now a considerable and growing literature on multi-dimensional measures of poverty and well-being. The literature is both theoretical and empirical and some of it is policy-oriented. Indeed, it is often argued that the multi-dimensionality of poverty has important implications at the policy level. The policy relevance of multi-dimensionality relates, in part, to the genuine possibility that an uni-dimensional approach to the measurement of well-being and poverty - such as that involved in some of the income-focussed poverty literature - is likely to underestimate the 'richness' or complexity of the nature of poverty, which needs to be addressed in any policy for poverty eradication. Furthermore, use of a multi-dimensional framework might actually alter the particular set of people who are identified as poor.

One response to arguments in favour of multi-dimensional approaches claims that, while multi-dimensional measures can be useful, they are usually *ad hoc*. Multi-dimensional measures typically involve some sort of weighting scheme or exercise to capture the relative importance of the different dimensions of poverty. Any ranking of countries or provinces which is based on such measures must then be highly sensitive to the specific weighting scheme adopted. There are various ways of responding to this challenge. Nonetheless, this issue is also relevant in the context of some uni-dimensional measures - such as those income- or expenditure-based metrics which use a basket of commodities, or a set of 'basic needs', to establish the income or expenditure poverty line. It is argued in this paper that while issues relating to sensitivity and robustness are not unique to multi-dimensional measures, they are easier to analyse in this context, because they *must* be explicitly addressed, and are more easily separated out in a multi-dimensional approach. It is argued, furthermore,

that there are numerous issues aside from than the specific issue of weighting which are relevant to the robustness of rankings based on these measures. In particular, there are issues relating to the choice of dimensions which are relevant, and to the choice of the ‘bottom line’ in terms of each dimension, in the analysis of poverty. These need to be distinguished from issues relating to weighting, if robustness is to be seriously examined.

In this paper, I address these issues in relation to certain standard methods for constructing multi-dimensional rankings - such as the Borda score and the Human Poverty Index (HPI) developed by the United Nations Development Programme (UNDP) - using data from the 1996 South African Census. The Census data have been the focus of some of the recent academic and policy-oriented literatures on South Africa. Some of the academic and policy debate has focussed on the inter-provincial picture of poverty and the distribution of poverty eradication grants to the various provinces. This paper focusses on inter-provincial rankings and it is clearly relevant to this debate.

The paper is organised as follows: in section 1, the various distinct issues raised by multi-dimensionality are discussed; in section 2, arbitrariness and robustness with respect to the choice of dimensions and weighting are examined; in section 3 issues relating to the choice of the ‘bottom line’ in each dimension are discussed; in section 4, the analysis is related to policy issues; and section 5 concludes.

### *1. Issues Raised by the Multi-Dimensionality of Poverty and Well-Being.*

There are numerous distinct issues relating to arbitrariness and robustness which are raised in the context of multi-dimensional measures of well-being and poverty. One can distinguish between issues which are: (i) dimension specific; and (ii) those which relate to the variety of dimensions. Many issues which are dimension specific - such as how to define a ‘bottom line’ in poverty evaluation - are closely related to those which arise in the income-

based uni-dimensional context. Nevertheless, some problems which arise because of the variety of dimensions involve formal and substantive issues which - while they may have been discussed in context of income poverty - often need to be dealt with in a more explicit manner in the multi-dimensional context.

To see how these problems arise it is worth considering the context in which they are being used. One central issue for poverty analysis is the identification of the poor. I term problems which relate primarily to this issue *identification problems*. Given the multi-dimensionality of poverty, there are numerous issues relating to both (i) and (ii) in relation to such problems. First, in relation to (ii), there is the choice of dimensions of well-being which are relevant to poverty analysis. Once this choice has been made, there is the further dimension specific issue of how much a person has to fall short in terms of each dimension to be poor in terms of that dimension - this is the issue of the 'critical level' at or below which one is poor in some dimension. In both cases, furthermore, it can be argued that there may be imprecision or 'fuzziness'. Vagueness about the 'bottom line' in each dimension - sometimes called 'vertical vagueness' - is distinct from vagueness about the dimensions of well-being which are relevant to the poverty evaluation exercise - 'horizontal vagueness' (Qizilbash, 2003). When researchers attempt to address these varieties of vagueness there is a further possibility of arbitrariness. For example, it can be argued that there is usually some arbitrariness in the selection of the upper and lower limits of the range of critical levels which involve ambiguous judgements (in the case of vertical vagueness). Issues relating to this problem have been raised in the fuzzy set theoretic poverty literature (Chiappero Martinetti 1994, 1997 and 2000 and Lelli, 2001). Once these issues have been settled, there is the further issue of how to define or identify someone as poor, taking account of all the dimensions of poverty and the various critical levels which have been identified (even when vagueness has

been allowed for in some way).

There are numerous approaches to dealing with this identification problem. Some take it that some individual (or household) is poor if she (or it) is poor in *any* dimension relating to poverty (Bourguignon and Chakravarty, 1997 and Brandolini and d'Alessio, 2001 *inter alia*). An alternative view of this problem would identify an individual (or household) as poor if she (or it) is poor in terms of *all* the specified dimensions. Yet another possibility is to classify people (households) as poor if they are poor in terms of some overall index or average of indices relating to poverty (e.g. Klasen, 1997 and 2000). In this last case, a further 'bottom line' is usually specified in terms of the average, or relevant overall index used. There is also the problem of the choice of weights which are assigned to specific dimensions if an overall index or average is used. These problems are clearly distinct in applications of multi-dimensional approaches. Finally, there is the relative importance of the *range* or number of dimensions in which some person or group is poor, and the extent of deprivation in some specific dimension. This contrast is sometimes characterised in terms of the contrast between the 'width' and the 'depth' of poverty (see Anand and Sen's note in UNDP, 1997). This is also a form of weighting problem, but it does not relate to the *relative* importance given to the different dimensions.

In most applications of multi-dimensional measures to identification problems only some of these issues are usually explicitly raised. So while most of the issues which are relevant to identification and weighting are clearly distinguished by Bourguignon and Chakravarty (1997), the issue of vagueness is not discussed. Furthermore, it is only rarely the case that *theoretical* arguments are presented in favour of the actual approach that is taken in response to issues raised by identification problems in the multi-dimensional context. So, for example, in applying the Amartya Sen's capability approach to the evaluation of poverty in

the South African context, Stephan Klasen uses a weighted average of deprivation indices in dealing with the identification problem without much justification for the use of an averaging approach (even though he argues that an alternative approach to selecting weights - using principal-component analysis - yields much the same result). There is also the issue of the dimensions actually used. Sometimes data on people's perceptions or evaluations are used to justify the choice of dimensions used in applying the particular approach (Klasen, 2000 and Clark and Qizilbash, 2002). In the absence of such arguments or data, it is often not clear whether the approach taken to the choice of dimensions is justified. It is certainly plausible to claim, in many cases, that the approach actually taken is *ad hoc*. In fact, often the best defence of the actual indicators selected relates purely to problems of data availability. This defence is sometimes presented in the context of the various indices developed by the UNDP.

While issues relating to identification problems are perhaps the most challenging issues that are raised by the literature on multi-dimensional poverty measurement, there are many contexts in which researchers or policy makers are not trying, specifically, to *identify* the poor using a multi-dimensional approach. Problems of the sort just discussed can arise in the context of national or provincial data which relate to how deprived people or households are in terms of particular dimensions. It is one such context of application - involving inter-provincial rankings - that I shall be concerned with in much of this paper. In this context, some group - such as those who are illiterate - has usually already been identified as deprived or poor in the specific dimension involved. Indeed, in this context the only data which is available is often dimension-specific.<sup>1</sup> The central issue is about how to arrive at a more general judgement or measure of poverty on the basis of such data. It is this problem which is involved in constructing some multi-dimensional poverty measures - such as the HPI developed by Sudhir Anand and Amartya Sen for the UNDP. In this case too, issues about the

choice of dimensions and the weights given to the specific dimensions, as well as issues relating to 'width' and depth', and to horizontal and vertical vagueness are relevant.

Related issues have clearly arisen in the context of uni-dimensional measures. So, for example, there has been the issue of whether to focus on just one dimension - nutrition - or some basket of 'basic needs' when estimating income poverty lines. There have been issues about the different weights attached to the components of poverty as well as about uncertainty about the precise location of the 'true poverty line' (Ravallion, 1994). Given a general concern with such issues, a sophisticated literature has grown which allows for many different income poverty lines (Atkinson, 1987, Foster and Shorrocks, 1988, Ravallion, 1994). However, given the focus on one dimension, most of the discussion surrounds the question of which poverty line - or which range of poverty lines - to choose in evaluation. Much of this literature also relates this issue to the question of whether to pick an 'absolute' or a 'relative' poverty line. It is complicated by further issues about the unit of accounting and the calculation for 'equivalent adults' when household data are being used to generate indices of individual poverty.

Given the problems posed by arbitrariness as well as issues relating to weighing, one approach to dealing with these problems has been to rely only on judgements which are invariant to all the possible choices of critical levels or dimensions or weights etc. used. This is the 'intersection' or 'dominance' approach - associated with Amartya Sen (1992, pp. 47-49) - which has been very influential. While it has had enormous influence on the academic literature, this approach has had less of an effect on the policy debate where specific weights and poverty lines are generally used. Indeed, in the policy context, Sen himself has been very active in developing new measures which are related to people's 'capabilities to live valuable lives' - using direct indices relating to the quality of life. The UNDP's measures of human

development and human poverty are examples of indices based on direct indices. These measures involve very specific weights and indices, which are potentially contentious. While certain attempts have been made to offer a qualified defence of the actual weights used in, and the dimensions selected for, these measures (Haq, 1995 and Anand and Sen, 2000 *inter alia*) issues about arbitrariness and robustness remain. Certain ‘fuzzy set theoretic’ measures have also been constructed using such direct indices of the quality of life (Chiappero Martinetti, 2000 and Qizilbash, 2002). These various measures can also be criticized on the grounds that they involve *ad hoc* judgements of the sort just described.

### *3. Horizontal Vagueness and Weighting.*

In this section, I focus on issues relating to the choice of dimensions and the relative importance given to the chosen dimensions, as well as the relative weight given to ‘depth’ and ‘width’. These issues are examined in the context of the UNDP’s framework for poverty measurement, particularly the HPI. This index is theoretically justified in terms of Sen’s ‘capability approach’ - which sees poverty in terms of the failure to have certain basic capabilities (Sen, 1993 and 1999). There are actually two variants of the HPI, one for developing countries and one for industrialised countries. The HPI for developing countries - HPI-1 - involves component indices relating to three dimensions which are thought of as related to ‘basic capabilities’ - health, education and a ‘decent standard of living’. Importantly it involves direct measures of the quality of life and is, for this reason, labelled an index of ‘human poverty’ rather than ‘income poverty’. Each component index used in the HPI-1 is either a headcount index or an average of headcount indices. The HPI-1 is actually a special case of a family of measures. I shall call this family the ‘Anand-Sen family of measures’, since it is based on the work of Sudhir Anand and Amartya Sen (UNDP, 1997). In defining this family of measures each component index used is termed a ‘shortfall’ and each shortfall

is indexed  $i$  so that  $S_i$  is the shortfall in terms of component  $i$ , for  $n$  dimensions, so that  $i=1,2 \dots n$ , and the weights attached to these components are written  $w_i$ . The weighted average of power  $\alpha$  is  $S(\alpha)$ :

$$(1) \quad S(\alpha) = \left\{ \frac{\sum_{i=1}^n w_i S_i^\alpha}{\sum_{i=1}^n w_i} \right\}^{1/\alpha}$$

In the case of the UNDP's HPI-1,  $n$  is three, equal weights are used and sum to one - so that each dimension is given a weight of  $1/3$  - and  $\alpha$  is set above 1. The motivation for setting  $\alpha$  above one is to allow for the 'depth' of a shortfall in terms of each component index to be picked up. An increase in a component index at a higher level of deprivation in that index will register more than at a lower level with this choice.  $\alpha$  is actually set at 3 of HPI-1. If it had been set at 1, (1) would reduce to an arithmetic average. There are numerous issues which are relevant to rankings based on the HPI-1. First, there is the issue of robustness to alternative choices of the set of dimensions of poverty, given horizontal vagueness about what counts as a 'basic capability' or 'basic need' as opposed to a 'non-basic' capability or need. Secondly, there is the question of how much rankings based on this index might change if we adjust the weights used. Third, there is the issue of the sensitivity of rankings to the choice of  $\alpha$ . There is, finally, a further question of how rankings based on variations of (1) compare with other multi-dimensional approaches to ranking. One popular alternative approach to ranking is the rank order method developed by the French mathematician Borda. This involves simply assigning a rank order score to each group in terms of each component index, and adding up the rank order scores to give the 'Borda score'. Unlike the Anand-Sen family of measures, this method relies exclusively on 'ordinal information' - because it uses rank order scores - so that changes in *levels* of poverty in specific dimensions have no impact on the measure if they leave rank orders in specific dimensions unchanged.

In addressing these issues, I focus on a specific context of application: the discussion of the inter-provincial ranking as regards poverty in South Africa in 1995-6. In this context, it is not obvious that the component indices used in the UNDP's HPI-1 are actually appropriate. The HPI-1 is used primarily for the purpose of *international* rankings of poverty, and nation specific priorities or issues may not be relevant for such rankings. However, in the South African context, there is a strong case for the inclusion of data relating to unemployment, which does not feature in the UNDP's HPI-1.<sup>2</sup> In fact, the indices which I focus on are ones which are listed in the publications which emerged from the 1996 South African Census. The use of some of these indices can be justified on the grounds that they relate to dimensions which are either prioritized or identified as components of a minimally adequate life by local South Africans (Klasen, 2000 and Clark and Qizilbash, 2002). Related indices are used in Stephan Klasen's attempt to apply the capability approach to the South African context.

The dimensions relate to: employment; health; access to clean water; shelter; education; energy use; and the ability to participate in the life of the community. As regards the 'bottom line' in terms of these indices, I shall make fairly arbitrary judgements in this section, and allow for different bottom lines in section 4. So in the case employment, the relevant index used is the rate of unemployment. In the case of water access, it is the proportion of the households whose access to water is from a dam, river, stream or spring. In the case of education, it is those individuals above twenty years of age with no schooling at all. In the case of health, there was no useful index in the Census publications, and an index relating to sanitation is used as a proxy measure. This index relates to the nature of a household's refuse removal, or lack of such removal. The index used is the proportion of the households without any refuse removal at all. In the case of shelter, the relevant index was the proportion of households living in traditional dwellings, informal housing (shacks etc.),

caravans and tents, as well as the homeless. An indicator relating to the energy used for cooking is also included: the proportion of households that use wood for cooking. Finally, one further index relating to engagement in social existence - and, in particular, the ability to communicate - is included: the proportion of households with no access to a telephone. This indicator might be justified in terms of considerations relating to 'social exclusion'. It might also be related to the ability to participate in the life of the community and to achieve self-respect. Of course, some might doubt that this indicator relates to 'basic capabilities' at all. Our intuitions about it might well be 'fuzzy', so that there is 'horizontal vagueness'.

In table 1, values for these indices are presented for each of the South African provinces. Before embarking on the specific ranking exercises, it is worth mentioning some claims in the related literature. In particular, Klasen (2000) has argued - using data for 1993 - that there were more households living in human (or capability) poverty in KwaZulu Natal, than would emerge from an examination of income alone. Qizilbash (2002) echoes this result using dimension specific data from 1996 Census publications on the basis of an inter-provincial ranking using Borda's method: the multi-dimensional ranking based on a selection of direct indices gives a quite different ordering of the provinces as compared to the standard expenditure measure quoted in the Census publications. In particular, KwaZulu Natal is amongst the worst three, while the Free State is amongst the three *best* in terms of the ranking. This result suggests a quite different ranking of the South African states as regards 'human poverty' to that based on 'expenditure poverty', since the Free State is usually amongst the worst provinces in terms of expenditure poverty measures, even when such expenditures are adjusted for household size (Leibbrandt and Woolard, 2000 and Ngwane *et al* 2001). Furthermore, in these studies KwaZulu Natal is not amongst the worst off states in terms of expenditure headcount indices of poverty. Qizilbash's result suggests that the income

and human poverty pictures are quite different. How robust is this result about the relative position of the Free State and KwaZulu Natal in human and expenditure poverty rankings?

In table 2, the Anand-Sen family of measures are given for values of  $\alpha$  equal to 1, 2 and 3. As with the HPI-1 equal weights are used, and set at 1/3. In the case where  $\alpha=1$ , we simply have an arithmetic average of the indices; in the case where  $\alpha = 3$ , we have a local equivalent of the HPI. In the case where  $\alpha = 2$ , 'depth' is given more importance than in the arithmetic average, but less importance than in the equivalent of the HPI. The measures are calculated both for the full list of indices, as well as for a subset of five indices, relating exclusively to education, employment, access to clean water, rubbish disposal and shelter. In the shorter list the indices relating to telephone access and energy use have been removed to allow for concerns relating to horizontal vagueness. The remaining indices relate to fairly uncontroversial indices: health, employment, clean water, education and shelter. It is noticeable that KwaZulu Natal does worse than the Free State in terms of the Anand-Sen family of measures. This is not surprising because it does worse in terms of each of the component indices used in the local variation on these measures used here. An elementary implication of this fact is that KwaZulu Natal must do worse than the Free State *whatever* the weights used. In this case, then, Sen's 'intersection' approach is useful and we have a robust ranking of the provinces which reinforces results in the existing literature.

What of the ranking of the remaining states? In all cases the worst three are: the Eastern Cape, Northern Province, and KwaZulu Natal. Northern Province does worse than the Eastern Cape when enough importance is given to 'depth' and all 7 indices are used. However, when the smaller list of indices is used, the Eastern Cape is worse than the Northern Province in terms of the family of Anand-Sen measures for the various chosen levels of  $\alpha$ . Nonetheless, the ranking might easily switch if more importance is given to those

with no schooling, since the Northern Province performs much worse than the Eastern Cape in terms of this index. As regards, the states which are doing best, the Western Cape, Gauteng, the Northern Cape and the Free State are invariably, respectively, first, second, third and fourth best. The position of the Free State is much better than in most expenditure-based rankings, though it is not third best (as in Qizilbash, 2002). For many of the states in the ‘middle’ of the ranking - Mpumalanga, North West, Northern Cape and the Free State - the values of the Anand-Sen family of measures are relatively ‘close’, suggesting that relatively small changes in weights would change the orderings. So the rankings of these states are not particularly robust.

How do these rankings compare with the Borda rankings based on the same indices? In table 3 the rankings based on the Borda ranking method are presented. Here the province which is doing worst (second worst, etc.) in terms of a particular index is given a rank order score of 9 (8 etc.). In the case of ties, if two provinces are doing worse than three (four etc.) provinces, they both get a rank order of four (five etc.). The sum of the rank order scores is the Borda score, and the ranking based on it is the Borda ranking. In the Borda ranking the same method is used in cases of ties.<sup>3</sup> The Borda score and the Borda ranking using all 7 indices, as well as that based on just 5 indices, are presented in table 3. The ranking of the provinces is much the same as that based on the Anand-Sen family of measures. The only difference between the Borda rankings using 7 indices rather than the subset of 5 is that the Free State and the Northern Cape are tied in ‘third best’ place if we use the full set of indices, while the Northern Cape beats the Free State to third best when one looks only at the subset of 5 indices. So the overall ranking of the states in terms of human poverty is fairly robust. In particular, KwaZulu Natal is third worst in terms of *all* the rankings just discussed.

#### *4. Vertical Vagueness.*

Thus far, I have abstracted from worries about robustness to how one defines the ‘bottom line.’ Vagueness about this ‘bottom line’ has been the focus of the fuzzy set theoretic poverty literature. In related work, Qizilbash (2002) used an approach due to Cheli and Lemmi (1995) to define the boundaries of the ambiguous zone (the zone of vagueness or ‘fuzziness’) in combination with data from the 1996 South African Census to rank the provinces of South Africa in terms of definite poverty. The Cheli and Lemmi approach attempts to respond to worries about arbitrariness in the context of vertical vagueness - particularly those associated with the measure developed by Cerioli and Zani (1990) - by only treating the worst off category for each dimension in the sample as definitely poor, and treating the best off group in the sample as definitely not poor. If one were to use this methodology one would have to amend at least one of the cut-offs used in section 3: that relating to the shelter indicator. Indeed, only those who are in the worst-off category in this dimension - i.e. the homeless - would count as definitely poor in this dimension on the Cheli and Lemmi methodology. In related work, Clark and Qizilbash (2002) also argue, on the basis of a recent survey on ‘The Essentials of Life’, that a not insignificant proportion of people interviewed in three locations in South Africa thought that someone could get by with just about any sort of dwelling or access to water. They conclude that if we are to define ‘bottom lines’ in terms of the views of ordinary South Africans, and to allow cut-offs as acceptable or ‘admissible’ if they are endorsed by a not insignificant proportion of South Africans, only those who have no access to water *at all* - even from a dam, stream etc. - are definitely poor in the dimension of water access.<sup>4</sup> Similarly, they conclude that only those with no dwelling - the homeless - are definitely poor in terms of shelter. In this case, their methodology echoes the Cheli and Lemmi methodology. The remaining cut-offs used in section 3 are consistent with both the Cheli and Lemmi methodology and the results reported by Clark and Qizilbash

(2002).

If we follow these suggestions and ‘toughen’ the ‘bottom lines’ used in conjunction with the indices from the 1996 Census, then only a tiny proportion (either zero or very close to zero) are definitely poor in the dimensions of shelter and access to water. On this basis, we might exclude indices relating to these dimensions in ranking the provinces in terms of ‘definite’ poverty. In line with the ‘tough’ stance taken with regard to the bottom line, we can also focus exclusively on the shorter list of five components of poverty used in section 3. Of these, water and shelter are excluded, so that only three indicators from the previous section remain: the proportion of the population above 20 years of age who have no schooling; the unemployment rate; and the proportion of households that have no rubbish disposal at all.

The values for the Anand-Sen family of measures for this subset of indices is given in table 4. The Borda scores and ranking based on these components is given in table 5. While the ranking based on the Anand-Sen family of measures is not very different from that in table 2, it is noticeable that Northern Province has now taken over from the Eastern Cape as the worst off province. KwaZulu Natal remains third worst as it did earlier, and is consistently worse than the Free State. The relative position of the Free State and the Northern Cape depends on the weight given to ‘depth’ (i.e. the choice of  $\alpha$ ). The results about the relative positions of the Northern Province, the Eastern Cape and KwaZulu Natal also emerge in the Borda ranking. In the Borda ranking, the Free State is fourth best, doing worse than the Northern Cape. So while the result in Qizilbash (2002) that the Free State is third best in the Borda ranking is not robust to the choice of indices and cut-offs used in the Borda ranking, the Free State is still far from being the worst province in terms of the poverty ranking (which is consistent with Qizilbash, 2002). This result, at least, is robust.

It is also worth noting some implications of ‘softening’ the bottom lines used. If one

takes such a 'soft stance' one would also no doubt allow all seven dimensions in table 1. So in table 6 headcount indices relating to all seven dimensions are included. In a number of cases, the 'cut-off' has been set less stringently than before. For example, in the case of education those who have begun, but not completed, primary education are included. In the case of energy used for cooking, all those who use dung for cooking are now included. In the case of water, those with access from a well or borehole are also included. Finally, in the case of access to a telephone, those who only have access to a phone at some distance from home are included. In the cases of shelter and employment, the same cut-offs as those in section 3 are used.

The arithmetic average of the indices - which is equivalent to setting  $\alpha=1$  for the Anand-Sen family of measures - is also presented in table 6. Again it is noticeable that the Northern Province is worse than the Eastern Cape and is the worst province in terms of this average. It is easy to check that this result is robust if we set  $\alpha=2$  or  $\alpha=3$  instead while using equal weights. Since the two provinces are relatively (if not very) 'close', nonetheless, the use of different weights might reverse the result. Finally, as with all the previous rankings KwaZulu Natal is third worst, though the Free State is third best. All but one of these results emerge in the Borda ranking based on these indicators. This is shown in table 7. The exceptional result is that the Free State is fourth best. Nonetheless, it is still true that the Free State is not amongst the three worst states as regards human poverty.

##### *5. Policy Relevance.*

These results are highly relevant to proposals relating to the allocation of poverty eradication funds to the provinces on the basis of poverty incidence. In recent work, Hirschowitz et al (2000) have argued for the use of two Statistics South Africa (SSA) indices - the household circumstances index and the household infrastructure index - in this context.

The SSA proposal is an attempt to alter the basis of the policy for distributing funds which focusses simply on household expenditures. Both SSA measures are multi-dimensional. The existing rationale for distribution clearly favours those provinces which tend to do badly in terms of expenditure poverty as opposed to human poverty. Indeed, as Qizilbash (2002) argues, the Free State would do considerably better on the basis of a policy which focusses on expenditure poverty than on one which focusses on a ranking using a multiplicity of dimensions and direct measures. Similarly, KwaZulu Natal would do considerably better on the basis of a policy which focusses on some multi-dimensional human poverty measures, than it would if the policy was based on expenditure headcount indices.

The various measures used in this paper - such as the Anand-Sen family of measures for various levels of  $\alpha$  - and the Borda score could be used to do much the same work that the SSA indices are meant to do. However, when the ranking of states is dependent on the choice of specific dimensions, cut-offs, or weights, there is likely to be controversy in the use of any specific formula which is used for the distribution of funds. In such cases, the particular decision which any government makes must be justified in some way. The discussion of robustness above may actually prove useful in this context.

The ranking of provinces in terms of the household circumstances and household infrastructure indices involve a number of stages (set out in Hirschowitz et al, 2000, pp. 74-80). In the first stage, a set of indicators is chosen (for household infrastructure and other concerns, respectively) on the basis of a principal-component analysis. The aim here is to include indices which are 'informative' and which are not closely correlated. In the second stage, the performance of provinces in terms of the chosen indices is judged using a rank order method of a particular sort for levels of deprivation. Specifically, performance in terms of each indicator was classified into various ranks order scores - with 3 being assigned to the

province with a relatively large proportion of the population living in the worst conditions, and 1 to a province with most of its population living in the best conditions, while provinces which fall in between are given a score of 2. The numbers which emerge from this exercise are then added up to give the 'interim score' for each province. The interim scores are then divided by the number of indices used to give an average interim score, and the average is then multiplied by the square root of population to give the relevant indices (SSA, 2000, pp. 76-77). The method is clearly a mixture of rank order scoring and an average using equal weights. It involves very specific cuts-offs and weights which are not justified. Any distribution of funds based on it might be highly controversial for this reason.

It is clear that much the same sort of exercise could be carried out using the Borda ranking or one of the Anand-Sen family of indices. However, as with the SSA measures some justification for the weighting, the choice of indicators and cut-offs used would need to be given. Equal weights can be justified in a limited way on the grounds that there is no reason to deviate from equal weights. However, they can also be justified, on the basis of a recent study by Clark and Qizilbash (2002) which finds that most of the central dimensions of poverty are given roughly equal weight on average in questionnaire responses from three locations in South Africa. Such questionnaire responses on a larger sample of South African households might certainly help to justify choices relating to weights, even though an average of marks can hide a wide disparity of responses.

In cases where the ranking is not robust to changes in the specific indices and cut-offs used, further justification is required. Where the ranking is 'close' on virtually all alternative measures, it seems perfectly sensible to give the relevant provinces the same level of funding per household. This might be the case, for example, with the Free State and the Northern Cape. It might also be the case as regards the relative position of Mpumalanga and the North

West. Alternatively, one might suppose that in such cases the larger share of funds per household might be allocated to the province with the most 'definite poverty' - i.e. those provinces which have the highest levels of poverty using the narrowest range of indices and the 'toughest' cut-offs. However, even in this 'tough' scenario, the poverty ranking between these provinces may not be robust. In the cases of Mpumalanga and the North West, the ranking depends on the amount of importance given to 'depth' when one looks at the Anand-Sen family of measures. In the case of the Free State and the Northern Cape, it depends on whether one looks at the Borda ranking or at the Anand-Sen family of measures.

As regards the better end of the poverty rankings, clearly the Western Cape is best, and Gauteng second best in terms of all ranking methods employed and ought to have the smallest allocations per capita.<sup>5</sup> At the bottom end of the ranking, KwaZulu Natal has the third worst position on *all* variations on the multi-dimensional measures, and should have the third highest per capita allocation. Finally, as regards the Eastern Cape and the Northern Province the ranking is not robust. One response to this would be to give both provinces the same funding per household. Another approach which the government might take would be to give more to the province with more definite poverty, when cut-offs have been defined in a 'tough' manner. On this basis, the largest amount of funds per household would go to the Northern Province rather than to the Eastern Cape. This would reverse the current ranking according to both household circumstances and household infrastructure, as presented by SSA (SSA, pp. 778-8). Giving more to the Northern Province than to the Eastern Cape on these grounds might also have a strong *political* rationale, relating to helping those who are definitely poor and thus unambiguously in need of help.

## 6. Conclusions.

There are several distinct issues which relate to arbitrariness and robustness involved

in measures and rankings involving a multi-dimensional poverty framework. In this paper, the focus has not been on identification problems. It has been on certain measures and inter-provincial rankings based on existing dimension specific data for South Africa in 1996. It turns out that the inter-provincial poverty rankings based on the Anand-Sen family of measures and the Borda scores are fairly robust. The results confirm claims in the related literature about the relative position of the Free State and KwaZulu Natal in inter-provincial rankings even when horizontal and vertical vagueness, as well as various weighting issues are allowed for. Even when the rankings are not robust, furthermore, the discussion suggests ways in which various allocations might be justified.

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## Notes.

1. Dutta, Pattanaik and Xu (forthcoming) discuss issues relating to linking this issue with the issue of identification in the multidimensional context.
2. It is included in the UNDP's HPI for developed countries.
3. For discussions of the Borda ranking method, see Dasgupta 1993 and Qizilbash, 1997 *inter alia*.
4. Clark and Qizilbash (2002) use 'at least 5%' as the crucial cut-off for a 'not insignificant' proportion of the sample they are concerned with.
5. Clearly, this argument only applies at the aggregate level and may need to be adjusted to allow for variations in the levels of poverty within provinces.